

Review of empirical fire spread models

- Formal methodologies for wind-slope correction

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Never Stand Still

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Overview

- Quasi-steady fire spread modelling
- Terrain-following coordinate system
- Methods for modelling the combined effects of wind and slope
- A general framework
- Counter-examples...



The quasi-steady ansatz

- It is assumed that a given set of environmental conditions *uniquely* determine the rate of spread of a fire, as well as other fire behaviour characteristics.
- Environmental conditions typically include: fuel load, structure and moisture content; air temperature and relative humidity; wind speed and direction; and topographic slope.
- Problems with the ansatz include:



Quasi-steady fire spread modelling

For example, the McArthur forest fire danger rating system:

$$R_w = 0.00153 F D^{0.987} \exp(0.0338T - 0.0345H + 0.0234w)$$

F is Fuel Weight (tonnes ha⁻¹)

D is Drought Factor (antecedent rainfall conditions)

T is Temperature (dry-bulb, °C)

H is Relative Humidity (%)

w is Wind Speed (average at height of 10m, km h⁻¹)

This gives the rate of spread of the fire, but in which direction will it spread?



Quasi-steady fire spread modelling

Consider a head-fire being blown directly upslope. In this case the fire will spread in the direction of the wind and slope at a speed determined as:

McArthur system (Australia):

$$R(w, \gamma_s) = R_w \exp(0.069\gamma_s) \quad \text{Multiplicative}$$

Rothermel system (U.S.):

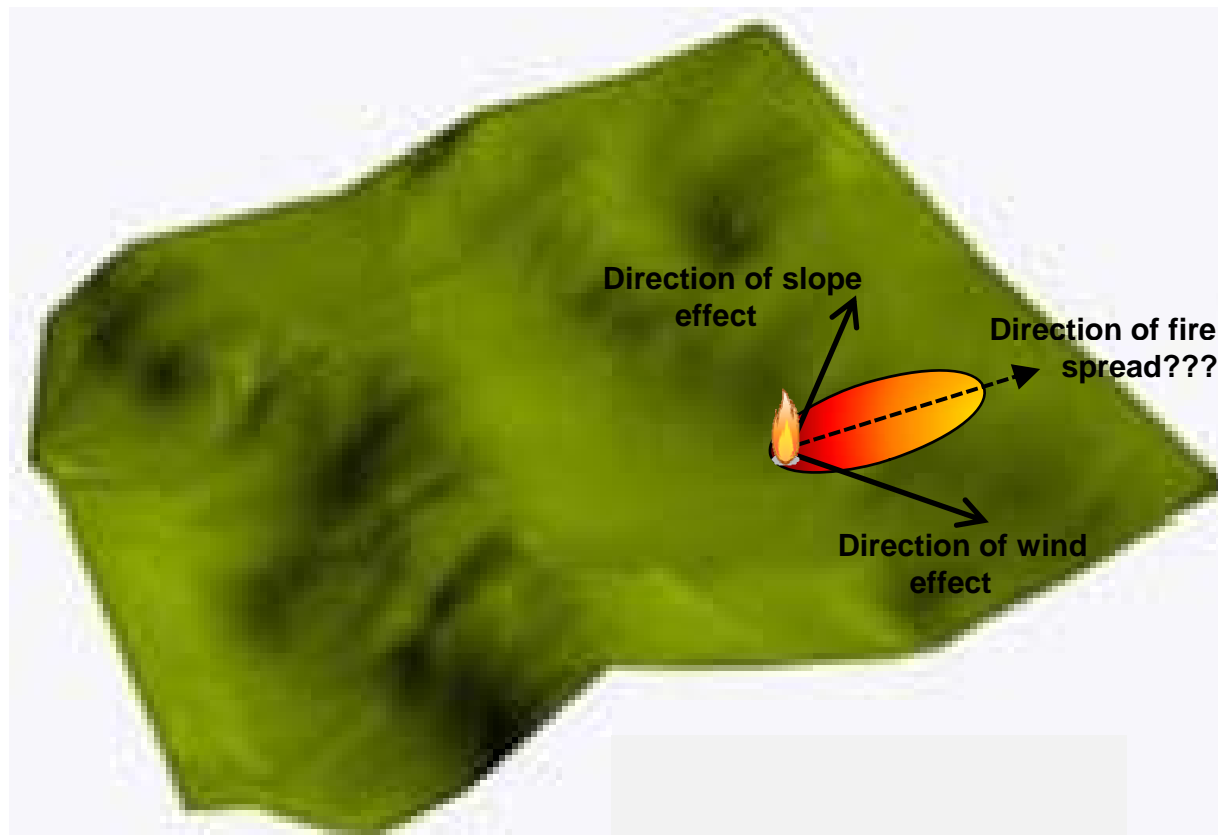
$$R(w, \gamma_s) = R_0 (1 + \phi_w + \phi_s) \quad \text{Additive}$$

where $\phi_s = A \tan^2 \gamma_s$ and $\phi_w = Cw^B$



Combining the effects of wind and slope

- But what if there is wind and slope present and they do not align?



Combining the effects of wind and slope

Geometric conventions:

Terrain elevation is described by the surface $h(x, y)$

The topographic gradient is $\|\nabla h\|$ and the topographic slope is given by

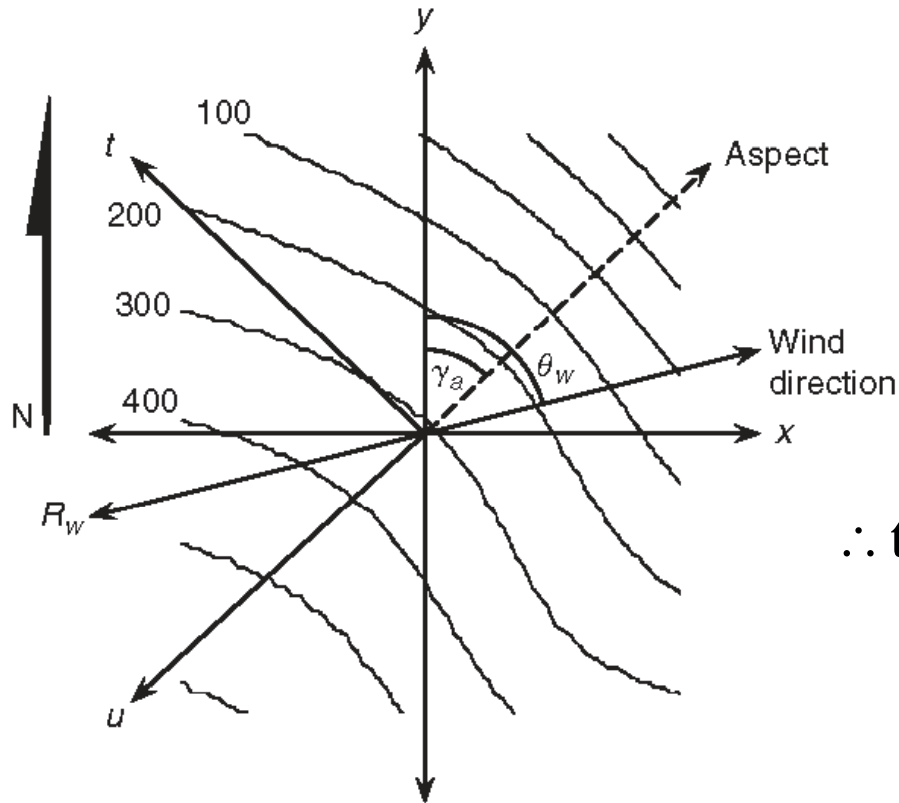
$$\tan \gamma_s = \|\nabla h\|$$

The effective topographic slope angle ψ in the direction of the wind-induced rate of spread vector \mathbf{R}_w is given by

$$\tan \psi = \nabla h \cdot \hat{\mathbf{R}}_w$$



Terrain-following coordinates



In these coordinates the unit vector in the direction of the wind-induced rate of spread vector is

$$\hat{\mathbf{R}}_w = \sin(\theta_w - \gamma_a) \hat{\mathbf{t}} + \cos(\theta_w - \gamma_a) \hat{\mathbf{u}}$$

$$\therefore \tan \psi = \nabla h \cdot \hat{\mathbf{R}}_w$$

$$= \left(\frac{\partial h}{\partial t} \hat{\mathbf{t}} + \frac{\partial h}{\partial u} \hat{\mathbf{u}} \right) \cdot \hat{\mathbf{R}}_w$$

$$= \frac{\partial h}{\partial t} \sin(\theta_w - \gamma_a) + \frac{\partial h}{\partial u} \cos(\theta_w - \gamma_a)$$

$$= \tan \gamma_s \cos(\theta_w - \gamma_a)$$



Combining the effects of wind and slope

The McArthur system.

Two options come to mind:

1. **Scalar method:** Use the standard slope correction formula with the slope angle replaced by the slope angle sensed in the direction that the wind is blowing.
2. **Vector method:** Vectorially decompose the wind-induced rate of spread vector into upslope and transverse components. Correct the upslope component for the effect of slope and then add the corrected upslope component to the transverse component



Combining the effects of wind and slope

The McArthur system.

1. Scalar method:

$$\mathbf{R}(w, \gamma_s) = \mathbf{R}_w \exp(0.069 \tan^{-1}(\tan \gamma_s \cos(\theta_w - \gamma_a)))$$

Note that the direction of spread is still in the direction of the wind. Only the magnitude has been corrected.

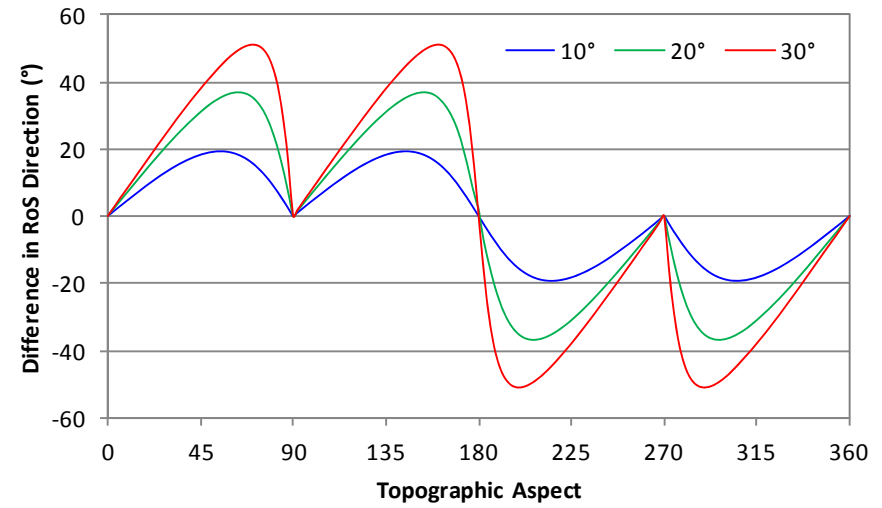
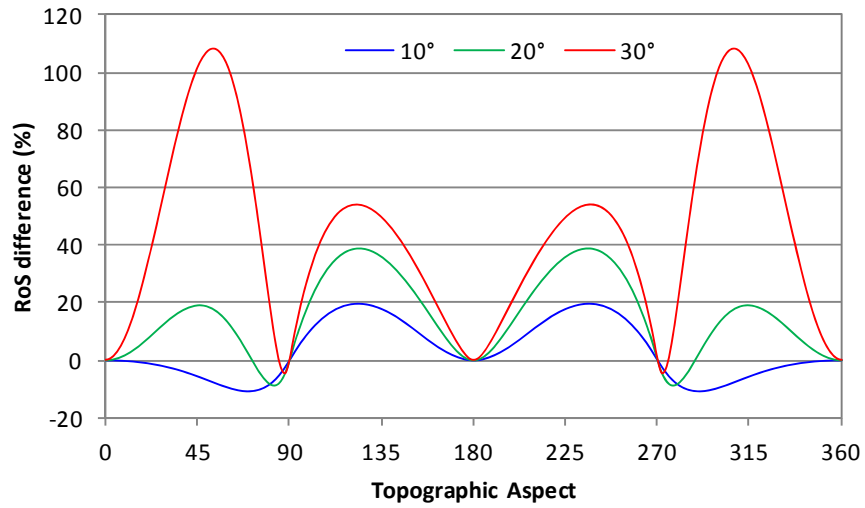
e.g. FIRESCAPE

2. Vector method:

$$\mathbf{R}(w, \gamma_s) = R_w \left[\sin(\theta_w - \gamma_a) \hat{\mathbf{t}} + \exp(0.069 \gamma_s) \cos(\theta_w - \gamma_a) \hat{\mathbf{u}} \right]$$



Differences in scalar and vector methods



Assuming that the wind is blowing from the north



Combining the effects of wind and slope

The Rothermel system.

1. Scalar method:

Note that the Rothermel method is a “whole of perimeter” method, so we consider the part of the fire perimeter travelling in the direction

$$\hat{\theta} = \sin \theta \hat{t} + \cos \theta \hat{u}$$

Now simply replace wind and slope in the ϕ_w and ϕ_s by their components in the direction of $\hat{\theta}$; that is:

$$R(\theta) = R_0 (1 + \phi_s(\theta) + \phi_w(\theta)) \quad \text{where} \quad \begin{aligned} \phi_s(\theta) &= A(\nabla h \cdot \hat{\theta})^2 \\ \phi_w(\theta) &= C(\mathbf{w} \cdot \hat{\theta})^B \end{aligned}$$



Combining the effects of wind and slope

The Rothermel system.

2. Vector method:

The vector $\hat{\mathbf{R}}_w = \sin(\theta_w - \gamma_a)\hat{\mathbf{t}} + \cos(\theta_w - \gamma_a)\hat{\mathbf{u}}$ points in the direction of the wind and so can be modified by ϕ_w directly.

The $\sin(\theta_w - \gamma_a)\hat{\mathbf{t}}$ part of the vector is not affected by slope and so we only need to modify the $\cos(\theta_w - \gamma_a)\hat{\mathbf{u}}$ part by ϕ_s .

The resultant vector, which describes the combined effects of wind and slope is

$$\mathbf{U}_{ws} = C\|\mathbf{w}\|^B \sin(\theta_w - \gamma_a)\hat{\mathbf{t}} + \left(C\|\mathbf{w}\|^B \cos(\theta_w - \gamma_a) + A \tan^2 \gamma_s \right) \hat{\mathbf{u}}$$



Combining the effects of wind and slope

The Rothermel system.

2. Vector method:

The length of \mathbf{U}_{ws} could be used in the Rothermel model in place of the term $\phi_w + \phi_s$ to give the magnitude of the rate of spread; that is,

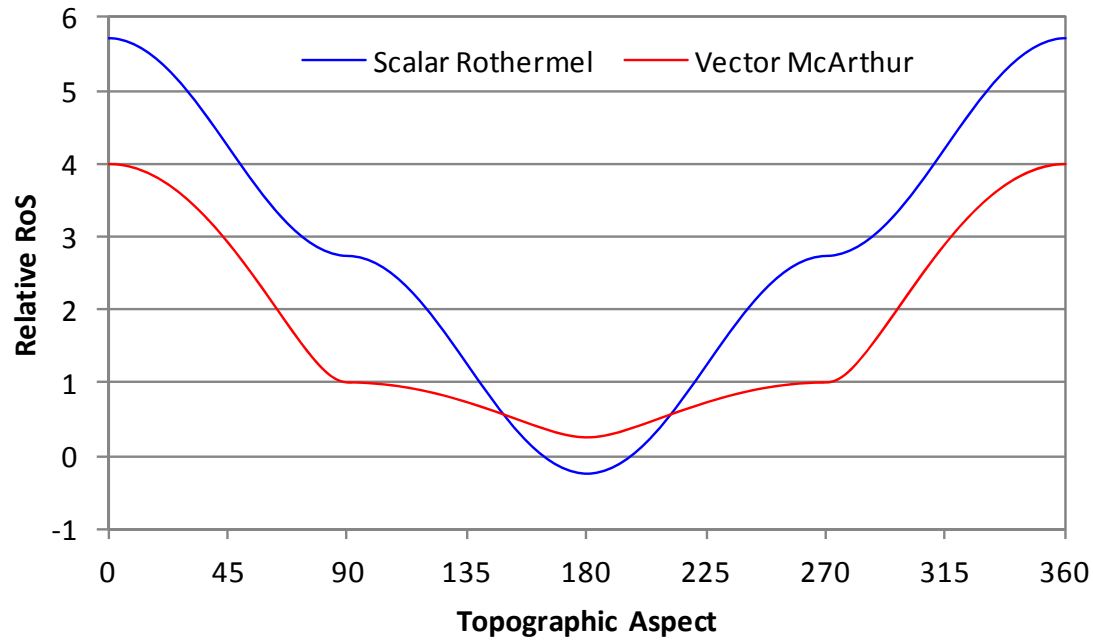
$$R(w, \gamma_s) = R_0 \left(1 + \|\mathbf{U}_{ws}\| \right)$$

With the direction of spread given by the direction of \mathbf{U}_{ws}

e.g. FARSITE



Combining the effects of wind and slope



The shapes are qualitatively similar!



A general framework for combining the effects of wind and slope...

Scalar methods for combining wind and slope effects fall into one of two classes: multiplicative and additive.

Multiplicative methods multiply the wind-induced rate of spread by a scalar slope correction factor:

$$R(w, \gamma_s) = \sigma(\gamma_s, \theta_w, \gamma_a) R_w$$

Multiplicative scalar methods give rise to vector methods of the form

$$\mathbf{R}(w, \gamma_s) = B_{-\gamma_a} S_{\gamma_s} B_{\gamma_a} \mathbf{R}_w$$

where...



A general framework for combining the effects of wind and slope...

$$B_{\gamma_a} = \begin{pmatrix} -\cos \gamma_a & \sin \gamma_a \\ -\sin \gamma_a & -\cos \gamma_a \end{pmatrix}$$

Change of basis matrix:

Changes from cardinal (x,y) coordinates to terrain-following coordinates (t,u)

$$S_{\gamma_s} = \begin{pmatrix} 1 & 0 \\ 0 & \sigma(\gamma_s, \theta_w) \end{pmatrix}$$

Slope-correction matrix:

Modifies the upslope component of rate of spread



A general framework for combining the effects of wind and slope...

Additive scalar methods are of the form:

$$R(w, \gamma_s) = \left(1 + \phi_{ws} \left(\|\mathbf{w}\|, \gamma_s, \theta_w, \gamma_a\right)\right) R_0$$

When we consider how to extend an additive scalar expression to a vector method, there appears to be at least two ways to proceed:

We already discussed replacing the scalar ϕ_{ws} with the norm of a vector like \mathbf{U}_{ws} :

$$R(w, \gamma_s) = R_0 \left(1 + \|\mathbf{U}_{ws}\|\right)$$

and direction given by the direction of \mathbf{U}_{ws} , or.....



A general framework for combining the effects of wind and slope...

Assign a direction to the base rate of spread R_0 and “vectorize” the entire factor that multiplies the base rate of spread:

$$\mathbf{R}(w, \gamma_s) = (\hat{\mathbf{n}} + \mathbf{U}_{ws})R_0$$

Unit normal vector to the fire perimeter

Wind-slope effect vector



Counter-examples

The under-pinning assumption of the existence of quasi-steady spread regime may not in fact be valid.

Instances of so-called *eruptive* fire spread seem to confirm this. In these instances fires can accelerate under conditions of constant slope, weather and fuel.

We did some simple experiments that demonstrated some more complex fire behaviour.

Indeed, slope appears to act as a bifurcation parameter in some instances.



Counter-examples



V-shaped canyon inclined at 30°



V-shaped canyon inclined at 40°



Counter-examples



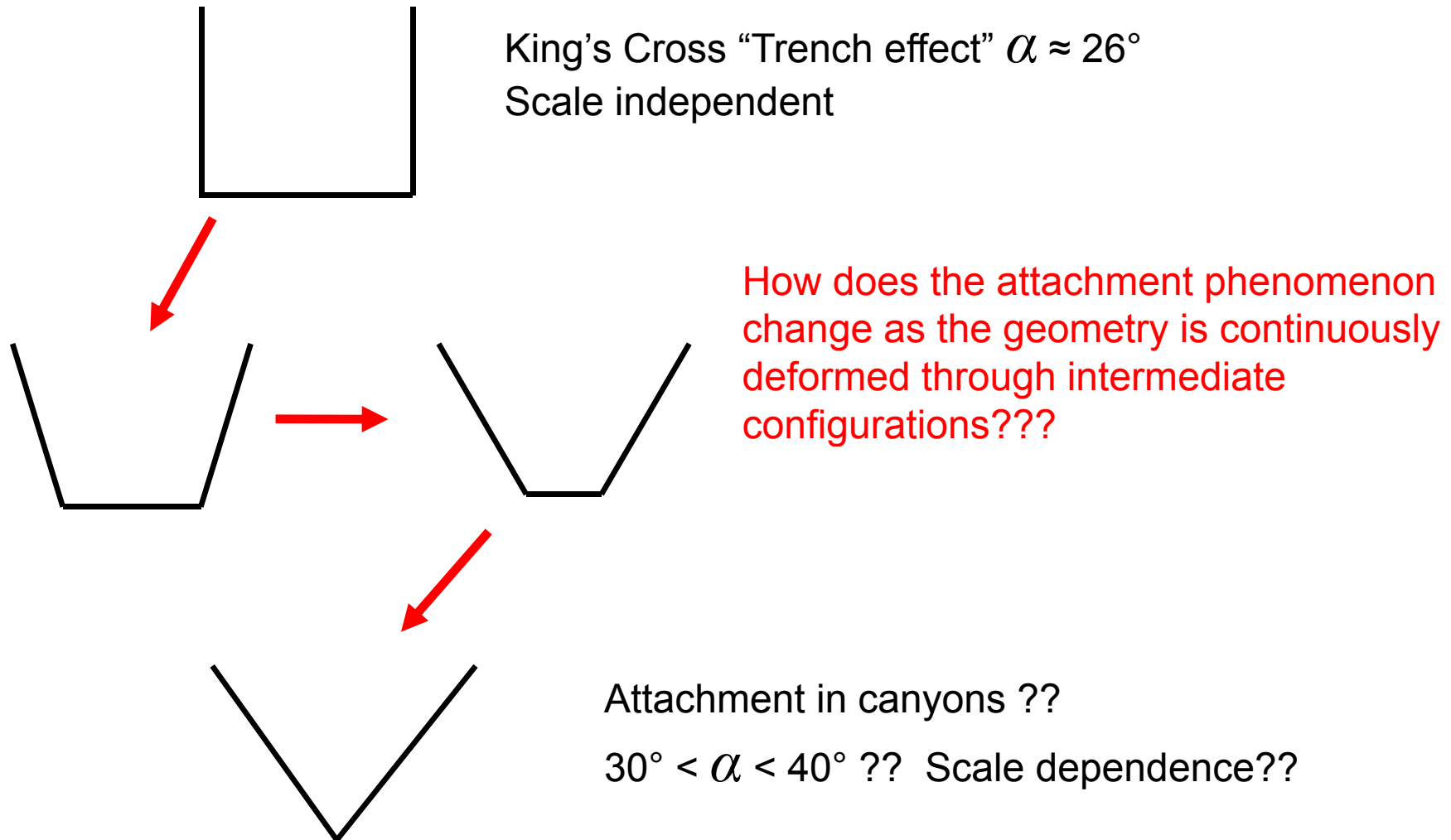
V-shaped canyon inclined at 30°



V-shaped canyon inclined at 40°



Future directions – Geometric attachment



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